

## Correction Examen 2021

Pb1 a)  $f(x) = \ln\left(\frac{6}{x^2-9}\right) = \ln(6) - \ln(x^2-9)$   
 $f'(x) = -\frac{2x}{x^2-9} = \frac{2x}{9-x^2}$

b)  $g'(t) = \frac{2 - \sqrt{t}}{2(\sqrt{t}-1)^2}$

Pb2 a)  $\lim_{x \rightarrow +\infty} \frac{\sqrt{2x^2-x} - x}{x - \sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{x(\sqrt{2 - \frac{1}{x}} - 1)}{x(1 - \frac{1}{\sqrt{x}})} = \sqrt{2} - 1$

b)  $\lim_{x \rightarrow 0} \left( \frac{1}{1-e^x} + \frac{1}{x} \right) = \lim_{x \rightarrow 0} \frac{x + 1 - e^x}{x(1 - e^x)}$

B-H des fois donne  $\lim_{x \rightarrow 0} \left( \frac{1}{1-e^x} + \frac{1}{x} \right) = \frac{1}{2}$

Pb3  $f(0) = b+1 = 0 \quad b = -1$

$$f'(x) = a - c e^{-cx}$$

$$f'(0) = a - c = 0 \quad a = c$$

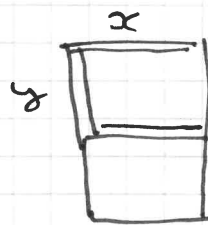
$$f''(x) = c^2 e^{-cx}$$

$$f''(0) = 4 \quad c^2 = 4 \quad c = 2 \quad (c > 0)$$

$$f(x) = 2x - 1 + e^{-2x}$$

Taylor d'ordre 3 :  $T_3(x) = 2x^2 - 4\frac{x^3}{3}$

Pb4



$1400 = 7x + 4y$  ~~Contrainte~~ Contrainte  
 $S = xy$   
 $S(y) = \frac{(1400-4y)y}{7} \quad y \in [0; 350]$   
max quand  $y = 175 \text{ m} \quad x = 100 \text{ m}$

Pb5 a)  $P^T P = \mathbb{I}_3$

b)  $P^{-1} = P^T$

c)  $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$

Pb6

$\left( \begin{array}{cccc|c} 1 & 0 & 4 & 0 & 1 \\ 0 & 1 & -2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right)$

$S = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1-4\lambda \\ -1+2\lambda \\ \lambda \\ 0 \end{pmatrix} \mid \lambda \in \mathbb{R} \right\}$