

Ex1 a) $f'(x) = \frac{1-4x}{2\sqrt{x}(1+4x)^2}$

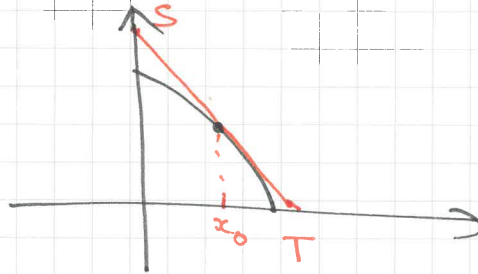
b) $g'(x) = \frac{1}{1+x^2}$

c) $h'(x) = e^x (1+x)$

Ex2 $y = p(x) = 4 - x^2$

$p'(x) = -2x$

$0 < x_0 < 4$



L: $y = p'(x_0)(x - x_0) + p(x_0)$

t: $y = -2x_0(x - x_0) + 4 - x_0^2$

T: $y = 0$

$0 = -2x_0(x - x_0) + 4 - x_0^2$

$x - x_0 = \frac{4 - x_0^2}{2x_0}$

$x = \frac{x_0^2 + 4}{2x_0}$

$x = x_0 + \frac{4 - x_0^2}{2x_0}$

$T\left(\frac{x_0^2 + 4}{2x_0}, 0\right)$

S: $x = 0$

$y = 2x_0^2 + 4 - x_0^2$

$= 4 + x_0^2$

$S(0, x_0^2 + 4)$

$A(x) = \frac{(x^2 + 4)^2}{2x}$

$0 < x < 4$

$A(x) = \frac{x^3}{2} + 4x + \frac{8}{x}$

$A'(x) = \frac{3x^4 + 8x^2 - 16}{2x^2} = \frac{3}{2x^2} \left(x^2 - \frac{4}{3}\right)(x^2 + 4)$

x	0	$\frac{2}{\sqrt{3}}$	4
$A'(x)$		-	+
$A(x)$			

* minimum pour $x = \frac{2}{\sqrt{3}}$

$$y = 4 - \frac{4}{3} = \frac{8}{3} \quad P\left(\frac{2}{\sqrt{3}}, \frac{8}{3}\right)$$

Ex3 a) $D(f) =]0, +\infty[$

b) $f'(x) = \frac{2}{x} (\ln(x) - 1)$

x	0	e	$+\infty$
$f'(x)$		-	+
$f(x)$			

c) $f''(x) = \frac{2(2 - \ln(x))}{x^2}$

x	0	e^2	$+\infty$
$f''(x)$		+	-
		∪	∩

d) pt d'inflexion d'abscisse $x = e^2$

Ex4 a) $\lim_{x \rightarrow 1} \frac{1-x^2}{\sin(\pi x)}$ $1-x = h \quad x \rightarrow 1 \quad h \rightarrow 0$

$$1-x^2 = (1-x)(1+x) = h(2+h)$$

$$\lim_{h \rightarrow 0} \frac{h(2+h)}{\sin(\pi(1-h))}$$

$$= \lim_{h \rightarrow 0} \frac{h(2+h)}{\sin(\pi - \pi h)} = \lim_{h \rightarrow 0} \frac{h(2+h)}{\sin(\pi h)}$$

$$= \lim_{h \rightarrow 0} \frac{h\pi (2+h)}{\sin(\pi h)} = \frac{2}{\pi}$$

$$b) \lim_{x \rightarrow 1} \frac{1-x + \ln(x)}{1 - \sqrt{2x-x^2}} = \frac{0}{0}$$

$$\begin{aligned} B-H \quad \lim_{x \rightarrow 1} \frac{-1 + \frac{1}{x}}{-\frac{(2-2x)}{2\sqrt{2x-x^2}}} &= \lim_{x \rightarrow 1} \frac{\frac{-x+1}{x}}{\frac{(1-x)}{\sqrt{2x-x^2}}} \\ &= \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{-\frac{1}{\sqrt{2x-x^2}}} = -1 \end{aligned}$$

$$g) \lim_{x \rightarrow -\infty} \frac{x^2}{\sqrt{x^2+1}} - \sqrt{x^2-x}$$

$$\begin{aligned} \frac{x^2}{\sqrt{x^2+1}} - \sqrt{x^2-x} &= \frac{x^2 \sqrt{(x^2-x)(x^2+1)}}{\sqrt{x^2+1}} \\ &= \frac{x^4 - (x^4 + x^2 - x^3 - x)}{\sqrt{x^2+1} (x^2 + \sqrt{x^4 - x^3 + x^2 + x})} \\ &= \frac{x^3 - x^2 + x}{\sqrt{x^2+1} (x^2 + \sqrt{x^4 - x^3 + x^2 + x})} \end{aligned}$$

$$= \frac{x^3 \left(1 - \frac{1}{x} + \frac{1}{x^2}\right)}{\sqrt{x^2+1} \left(1 + \sqrt{1 - \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}}\right)}$$

$$= \frac{1 - \frac{1}{x} + \frac{1}{x^2}}{\sqrt{1 + \frac{1}{x^2}} \left(1 + \sqrt{1 - \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}}\right)}$$

$$n < 0$$

$$= \frac{1 - \frac{1}{x} + \frac{1}{x^2}}{\sqrt{1 + \frac{1}{x^2}} \left(1 + \sqrt{1 - \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}}\right)}$$

$$= \frac{1 - \frac{1}{x} + \frac{1}{x^2}}{\sqrt{1 + \frac{1}{x^2}} \left(1 + \sqrt{1 - \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}}\right)}$$

$$\lim_{x \rightarrow -\infty} \frac{x^2}{\sqrt{x^2+1}} - \sqrt{x^2-x} = -\frac{1}{2}$$