

Ex1

Correction TE2 - MAT1-B 23-24

$$a) f'(x) = \frac{(\cos(x) - \sin(x))(\sin(x) - \cos(x)) - (\sin(x) + \cos(x))(\cos(x) + \sin(x))}{(\sin(x) - \cos(x))^2}$$

$$= - \frac{2}{(\sin(x) - \cos(x))^2}$$

$$b) g(x) = (x(x \cdot x^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}} = x^{\frac{7}{8}}$$

$$g'(x) = \frac{7}{8} x^{-\frac{1}{8}}$$

$$c) h(u) = \arctan\left(\frac{1}{u}\right)$$

$$h'(u) = \frac{-\frac{1}{u^2}}{1 + \frac{1}{u^2}} = \frac{-\frac{1}{u^2}}{\frac{u^2 + 1}{u^2}} = -\frac{1}{1 + u^2}$$

$$d) L(t) = \frac{t + \ln(t)}{(t+1)^2} \quad L'(t) = \frac{\left(1 + \frac{1}{t}\right)(t+1) - (t + \ln(t))}{(t+1)^2}$$

$$L'(t) = \frac{(t+1)^2 - t(t + \ln(t))}{t(t+1)^2} = \frac{t^2 + 2t + 1 - t^2 - t \ln(t)}{t(t+1)^2}$$

$$= \frac{1 + 2t - t \ln(t)}{t(t+1)^2}$$

Ex2 a) $\lim_{x \rightarrow +\infty} \frac{(8+x^2)^{\frac{1}{2}}}{x^2+x} = \lim_{x \rightarrow +\infty} \frac{x^{\frac{2}{3}} \left(\frac{8}{x^2} + 1\right)^{\frac{1}{3}}}{x^2 \left(1 + \frac{1}{x}\right)}$

$$= \lim_{x \rightarrow +\infty} \frac{x^{-\frac{4}{3}} \left(\frac{8}{x^2} + 1\right)^{\frac{1}{3}}}{1 + \frac{1}{x}} = 0$$

$$b) \lim_{x \rightarrow 1} \frac{1-x^2}{\sin(\pi x)} = \frac{0}{0}$$

$$\text{B-H} \quad \lim_{x \rightarrow 1} \frac{-2x}{\pi \cos(\pi x)} = \frac{-2}{\pi(-1)} = \frac{2}{\pi}$$

$$c) \lim_{x \rightarrow 0} \frac{x \cos(x) + 2 \ln(1+x)}{x} = \frac{0}{0}$$

$$\text{B-H} \quad \lim_{x \rightarrow 0} \frac{\cos(x) - x \sin(x) + \frac{2}{1+x}}{1} = \frac{1-0+2}{1}$$

$$= \lim_{x \rightarrow 0} = 3$$

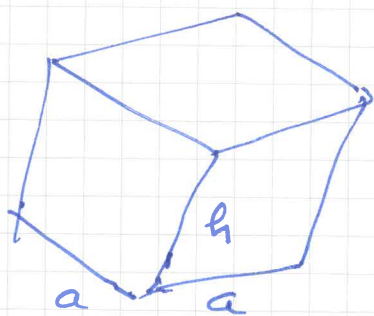
$$d) \lim_{x \rightarrow -\infty} 2x + \sqrt{4x^2 + x + 1} = \infty + \infty$$

$$\lim_{x \rightarrow -\infty} \frac{4x^2 - (4x^2 + x + 1)}{2x - \sqrt{4x^2 + x + 1}} = \lim_{x \rightarrow -\infty} \frac{-x - 1}{2x - \sqrt{4x^2 + x + 1}}$$

$$= \lim_{x \rightarrow -\infty} \frac{-1 - \frac{1}{x}}{2 + \sqrt{4 + \frac{1}{x} + \frac{1}{x^2}}}$$

$$= \frac{-1}{4}$$

Ex3



$$V = a^2 h \quad h = \frac{V}{a^2}$$

$$S = a^2 + 4ah \quad a > 0$$

$$S(a) = a^2 + \frac{4V}{a} = \frac{a^3 + 4V}{a} = \frac{a^3}{a} + \frac{4V}{a}$$

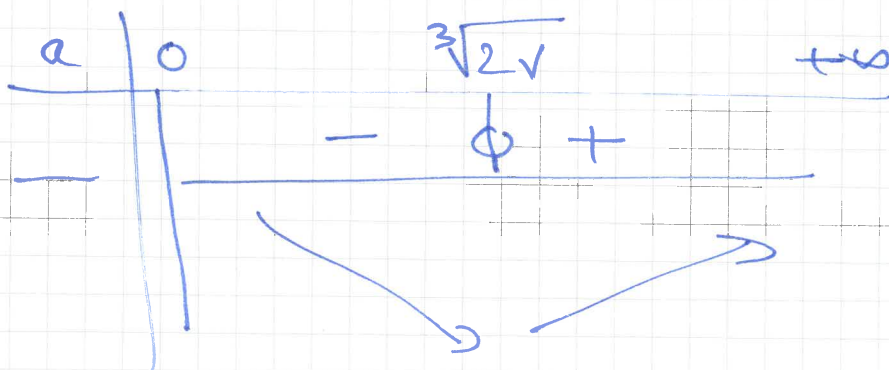
$$S'(a) = 2a - \frac{4V}{a^2} = \frac{2a^3 - 4V}{a^2}$$

$$S'(a) = 0$$

$$a = \sqrt[3]{2V}$$

$$a^3 = 2V$$

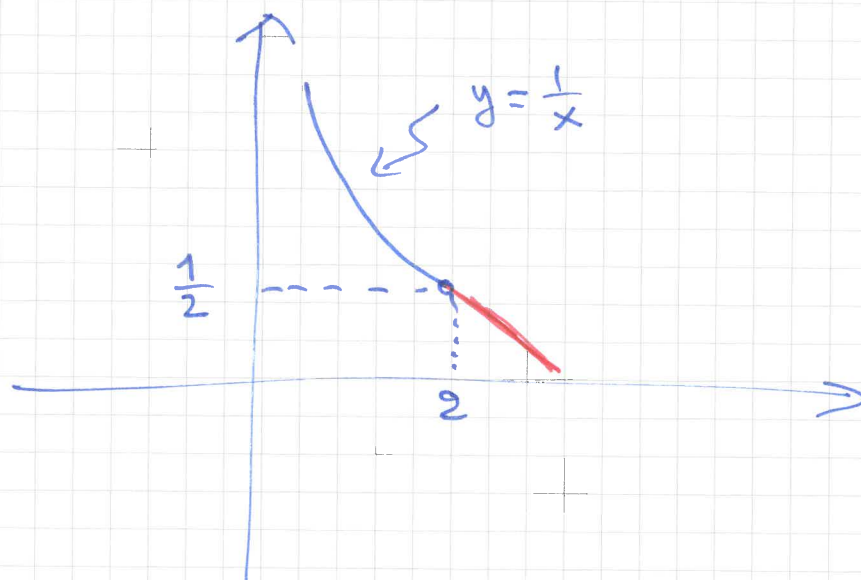
$$h = \frac{V}{a^2} = \frac{\frac{a^3}{2}}{a^2} = \frac{a}{2}$$



on a un minimum pour $a = \sqrt[3]{2V}$

dans ce cas on a $h = \frac{a}{2}$

Ex4 a) $f(x) = \frac{1}{x} \quad x \in]0, 2]$



$$f'(x) = -\frac{1}{x^2}$$

$$f'(2) = -\frac{1}{4}$$

c: $y = f'(2)(x-2) + \frac{1}{2} = -\frac{1}{4}x + 1$

on prolonge la fonction par la tangente!

b) $f(x) = x^2 \ln(x) \quad x > 0$

$$f'(x) = 2x \ln(x) + x$$

$$f''(x) = 2 \ln(x) + 2 + 1 = 2 \ln(x) + 3$$

$$f''(x) = 0$$

$$2 \ln(x) + 3 = 0$$

$$\ln(x) = -\frac{3}{2}$$

$$x = e^{-\frac{3}{2}}$$

$$e^{-3/2}$$

$+\infty$

x	0	$e^{-3/2}$	$+\infty$
$f''(x)$		-	+
$f(x)$		Concave	Convexe

$$x = e^{-\frac{3}{2}}$$

pt d'inflexion